Note on Diamond (1984):

Following Diamond (1984), consider a framework in which $n$ identical firms seek to finance projects, each firm requires an investment of one unit of account, and the returns of each firm are identically independently distributed. The cash flow $V$ that a firm obtains from its investment is a priori unobservable to lenders. This gives rise to a moral hazard problem that can be solved by monitoring the firm (at a cost of $K$) or by designing a debt contract characterized by a cost $D$. Assume also that there are small each investors $(1/m)$ in the market so that $m$ of the are needed for financing one project. For simplicity assume that the total number of investors is at least $mn$ so that all projects can be financed.

If a bank ($FI$) emerges, it can choose to monitor each firm (total cost $nK$) or to sign a debt contract with each of them (total cost $nD$). Since $K < D$, the first solution is preferable: the bank is therefore a delegated monitor, which monitors borrowers on behalf of lenders. However, there is question that who monitors the monitor. Direct monitoring would be clearly inefficient, so the only solution is that the bank offers a debt contract or deposit contract under which each investor is promised a nominal amount $iD/m$ in exchange for a deposit $1/m$, and the bank is liquidated if its announced cash flow is less than $niD$. This contract should be incentive compatible in the sense that bank has an interest in sending the truthful declaration so that the realized cash flow from the firm ($\sum_{i=1}^{n} V_i - nK$). The equilibrium level of $iD$ (which represents the nominal rate of return on deposits) and the cost of delegation will depend on $n$. Assume that depositors are risk neutral and have access to an outside investment alternatives yielding a gross expected return of $R$. The equilibrium repayment on deposits, $iD$, is determined by

$$E[\min(\sum_{i=1}^{n} V_i - nK, n iD)] = nR,$$

which expresses that the expected unit return on risky deposits equals $R$.

The total cost of delegation, $D_n$ is equal to the expectation of the penalty in case of bankrupts or bank failure:

$$D_n = E[\max(n iD + nK - \sum_{i=1}^{n} V_i, 0)].$$
Delegated monitoring is more efficient than direct lending if and only if

\[
\frac{nK + D_n}{n} < \frac{nmK}{n}.
\]

(3)

delegated monitoring cost \quad direct lending cost

Result by Diamond 1984: If monitoring is efficient (less costly), investors are small and investment is profitable \((E(V) > K + R)\), financial intermediation (delegated monitoring) dominates direct lending as soon as \(n\) is large enough (diversification).

Proof: Condition (3) must be proved. Dividing it by \(n\) yields an equivalent form:

\[
K + \frac{D_n}{n} < mK.
\]

Since \(m > 1\), it is enough to prove that \(D_n/n\) goes to 0 toward \(n \to \infty\). Dividing equations (1) and (2) by \(n\) yields

\[
E[\min\left(\sum_{i=1}^{n} \frac{1}{n} V_i - K, i_D\right)] = R,
\]

(4)

and

\[
\frac{D_n}{n} = E[\max(i_D + K - \frac{1}{n} \sum_{i=1}^{n} V_i, 0)].
\]

(5)

Strong law of large numbers dictates that \(\frac{1}{n} \sum_{i=1}^{n} V_i\) converges almost to \(E(V)\). Since \(E(V) > K + R\), relation (4) shows that \(\lim_{n \to \infty} i_D = R\) (i.e. deposits are asymptotically riskless). Therefore, by (5)

\[
\lim_{n \to \infty} \frac{D_n}{n} = \max(R + K - E(V), 0) = 0.
\]