Most firms produce or sell more than one product; for example Braun produces Personal Care Appliances such as electric shavers, hair dryers, electric toothbrushes etc or Braun produces Households Appliances such as stream irons, toasters, coffee makers, hand blenders etc. So, in order to maximize company’s profit, the levels of outputs and prices for the related commodities must be determined jointly. Rule: When a firm produces two products, \( X \) and \( Y \) that are related in consumption either as substitutes or complements, the manager of the multi-product firm maximizes profit by producing and selling the amounts of \( X \) and \( Y \) for which

\[
MR_X = MC_X \\
MR_Y = MC_Y
\]

are simultaneously satisfied. The profit maximizing prices, \( P_X \) and \( P_Y \) are determined by substituting the optimal levels of \( X \) and \( Y \) into the demand functions.

EXAMPLE: Suppose the company produces two types of automobile vacuum cleaners. One which we denote as product \( X \), plugs into the cigarette lighter receptacle; the other-product \( Y \)-has rechargeable batteries. Assuming that there is no relation between the two goods other than the apparent substitutability in consumption, the manager wanted to determine the profit maximizing levels of production and price for the two products. Assume that the demand functions are as follows

\[
P_X = 70 - 0.0005Q_X - 0.00075Q_Y \quad \text{and} \quad P_Y = 80 - 0.001Q_Y - 0.0005Q_X
\]

The marginal revenue functions are from \( TR_X = P_X Q_X = (70 - 0.0005Q_X - 0.00075Q_Y)Q_X \),

\[
MR_X = 70 - 0.001Q_X - 0.00075Q_Y
\]

and from \( TR_Y = P_Y Q_Y = (80 - 0.001Q_Y - 0.0005Q_X)Q_Y \),

\[
MR_Y = 80 - 0.002Q_Y - 0.0005Q_X
\]

As you notice \( MR_X \) is a function of \( Q_X \) and \( Q_Y \), as in \( MR_Y \). Moreover, the production manager obtained the marginal cost functions as

\[
MC_X = 10 + 0.004Q_X \quad \text{and} \quad MC_Y = 20 + 0.00025Q_Y
\]
To determine the output that will maximize profit, the manager equates $MR$ and $MC$ for the two products:

\[ 70 - 0.001Q_X - 0.00075Q_Y = 10 + 0.004Q_X \]
\[ 80 - 0.002Q_Y - 0.0005Q_X = 20 + 0.00025Q_Y \]

Solving these equations simultaneously for $Q_X$ and $Q_Y$, the profit maximizing outputs will be $Q^*_X = 30000$ and $Q^*_Y = 20000$. Finally, the prices are

\[ P^*_X = 70 - 0.0005(30000) - 0.00075(20000) = $40 \]
and
\[ P^*_Y = 80 - 0.001(20000) - 0.0005(30000) = $45. \]