EXAMPLE: Transfer Pricing. Suppose Y-Motors has the following demand for its automobiles:

\[ P = 20000 - Q \quad \text{(and} \quad MR = 20000 - 2Q) \]

and total cost as

\[ TC(Q) = 8000Q \quad \text{(and} \quad MC = 8000.) \]

There is upstream division of Y-Motors which produces engines with the following cost

\[ TC(Q_E) = 2Q_E^2 \quad \text{(and} \quad MC_E = 4Q_E.) \]

a. Suppose there is no outside market for the engines. How many engines and cars should the firm produce, and what should the transfer prices for engines be?

To solve this problem, we set the net marginal revenue for engines equal to the marginal cost of producing engines.

\[ NMR_E = MR - MC = MC_E = 20000 - 2Q - 8000 = 12000 - 2Q_E = 4Q_E \]

\[ Q_E^* = 2000 \]

The firm should produce \( Q_E = 2000 \) and 2000 cars. The optimal transfer price is the marginal cost of these 2000 engines: \( P_E = MC_E = 4Q_E = 4(2000) = $8000 \).

b. Now suppose that engines can be bought or sold from $6000 in an outside competitive market. This is below the $8000 transfer price that is optimal when there is no outside market. To find optimal quantity of engines, set \( NMR_E = $6000 (= \text{outside price}) \)

\[ NMR_E = 12000 - 2Q_E = $6000 \quad \rightarrow \quad Q_E^* = 3000. \]

Now the company produces more engines because its cost of engines is lower. Also, since the transfer price for engines is now $6000, the Engine Division supplies only 1500 engines:

\[ MC_E(Q_E = 1500) = 4Q_E = 4(1500) = $6000 \quad \rightarrow \quad Q_E^* = 1500 \]
and the remaining 1500 engines are bought in the outside markets.

c. Now suppose Y-Motors has some market power on their engines with the following outside demand:

\[ P_{E,O} = 10000 - Q_E \quad \text{(and)} \quad MR_{E,O} = 10000 - 2Q_E \]

where \( P_{E,O} \) is the price in the outside market for engines. To determine the optimal transfer price, we have to find the total net marginal revenue by horizontally summing \( MR_{E,O} \) and \( NMR_E \) as

\[ MR_{E,O} = 10000 - 2Q_E \rightarrow Q_E = 5000 - 0.5MR_{E,O} \]
\[ NMR_E = 12000 - 2Q_E \rightarrow Q_E = 6000 - 0.5NMR_E. \]

The horizontal summation yields

\[ Q_E = 11000 - NMR_{E,Total} \rightarrow NMR_{E,Total} = 11000 - Q_E. \]

Now set the total net marginal revenue to marginal cost of producing engines:

\[ NMR_{E,Total} = 11000 - Q_E = MC_E = 4Q_E \rightarrow Q^*_E = 2200. \]

How many of these engines should go to the assembled car division, and how many to the outside market? To answer these questions, we have to equate marginal cost of producing these 2200 engines (\( MC(Q_E = 4Q_E = 4(2200) = 8800) \)) to the marginal revenue from outside market and marginal revenue from transferred division:

\[ 8800 = MR_{E,O} = 10000 - 2Q_E \rightarrow Q^*_E = 600. \]

Therefore, 600 engines should be sold in the outside market and at the transfer price of $8800, assembled car division gets

\[ 8800 = NMR_E = 12000 - 2Q_E \rightarrow Q^*_E = 1600. \]

So, 1600 engines should be supplied to the inside division in order to be used in the production of cars.