Solutions for Exercise III (Note: There may be some typing errors. Be careful!)

1. a.  

<table>
<thead>
<tr>
<th>Units</th>
<th>Firm 1</th>
<th></th>
<th>Firm 2</th>
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<td>$MC_1$</td>
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<td>$AC_2$</td>
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<td>-</td>
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b. The cartel should assign production such that the lowest marginal cost is achieved for each unit:

<table>
<thead>
<tr>
<th>Cartel Unit</th>
<th>Assigned Firm</th>
<th>$MC$</th>
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<tr>
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<td>4</td>
<td>$18$</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>$20$</td>
</tr>
</tbody>
</table>

Therefore Firm 1 and 4 produces 2 units each and Firm 2 and 3 produces 3 units each.

c. At this level of output, Firm 2 has the lowest marginal cost for producing one more unit beyond its allocation ($MC = \$21$ for the fourth unit for Firm 2). In addition $MC = \$21$ is less than the price of $\$25$. For all other firms, the next unit has a marginal cost equal to or greater than $\$25$. Firm 2 has the most incentive to cheat, while Firms 3 and 4 have no incentive to cheat, and Firm 1 is indifferent.

2. a. To determine the Cournot-Nash equilibrium, we first write the profit
functions,
\begin{align*}
\pi_1 &= (150 - (Q_1 + Q_2)Q_1 - 30Q_1 \\
\pi_2 &= (150 - (Q_1 + Q_2))Q_2 - 30Q_2
\end{align*}
then, derive the first-order-conditions,
\begin{align*}
\frac{\partial \pi_1}{\partial Q_1} &= 120 - 2Q_1 - Q_2 = 0 \\
\frac{\partial \pi_2}{\partial Q_2} &= 120 - 2Q_2 - Q_1 = 0
\end{align*}
and solve them simultaneously. Thus, \( Q_1^* = 40 \) and \( Q_2^* = 40 \). \( P = 150 - (40 + 40) = $70 \). Profits are \( \pi_1 = (40x$70) - (30x40) = $1600 = \pi_2 \).

b. First, write cartel’s supply by horizontal summation of \( MC \). Since \( MC \) are constant at $30. The supply function would also be constant so,
\[
\frac{150 - 2Q}{MC} = 30 \rightarrow Q^* = 60.
\]
Thus, \( P = 150 - 60 = $90 \) and \( MR = 150 - 2(60) = $30 \). Each identical firm produces half of 60 units. And, \( \pi_1 = ($90x30) - (30x30) = $1800 = \pi_2 \). The cartel profit is \( \pi = \pi_1 + \pi_2 = $3600 \).

c. If the Firm 1 were the only firm, again from \( MR = MC \), we have the same result as in part (b) but now, \( Q_1^* = 60, P = $90 \) and \( \pi_1 = $3600 \).

3. a. 
\[
\begin{align*}
\frac{MR_e}{108 - 2Q_e - 2} = \frac{MC_e}{6 + 2Q_e} \\
106 - 2Q_e &= 6 + 2Q_e \\
25 &= Q_e^*
\end{align*}
\]
and, transfer price, \( P_t = MC_e = 6 + 2(25) = $56 \).

Thus the firm will produce 25 digital radios.

b. First we solve for the profit maximizing level of inside and outside sales (Note: \( Q_{el} \) denotes inside sale whereas \( Q_{eo} \) is used to represent outside sales),
\[
\pi = (108 - Q_{el})Q_{el} - (30 + 2Q_{el}) + (72 - 1.5Q_{eo})Q_{eo} \\
- (70 + 6(Q_{el} + Q_{eo}) + (Q_{el} + Q_{eo})^2)
\]
Profit maximization implies:
\begin{align*}
\frac{\partial \pi}{\partial Q_{el}} &= 108 - 2Q_{el} - 2 - 6 - 2(Q_{el} + Q_{eo}) = 0 \\
\frac{\partial \pi}{\partial Q_{eo}} &= 72 - 3Q_{eo} - 6 - 2(Q_{el} + Q_{eo}) = 0
\end{align*}
which yields

\[
\begin{align*}
25 &= Q_{eI} + 0.5Q_{eO} \\
66 &= 5Q_{eO} + 2Q_{eI}
\end{align*}
\]

and \(Q_{eO} = 4\) and \(Q_{eI} = 23\). Thus the total component will be \(Q_e = Q_{eO} + Q_{eI} = 4 + 23 = 27\), and marginal cost of component is \(MC = 6 + 2Q_e = \$60\). This value is also transfer price, \(P_t = MC_L = \$60\) for profit maximizing electronic division. The outside price would be \(P = 72 - (3/2)4 = \$66\). Since the firm has market power in external market \(P\) is higher than the \(MC = \$60\). (Check also that \(NMR = 106 - 2(23) = MC = \$60\).)

4. Consider a firm that produces using two plants, \(A\) and \(B\), with the following marginal cost functions:

\[
\begin{align*}
MC_A &= 10 + 0.01Q_A \\
MC_B &= 4 + 0.03Q_B
\end{align*}
\]

If the manager of this firm wishes to produce 1400 units at the least possible total cost, should 700 units be produced in each plant? Why or why not? If not, what should the allocation be?

5. First, find \(MR_X = 20 - 0.2Q_X - 0.05Q_Y\) and \(MR_Y = 70 - 0.6Q_Y - 0.1Q_X\) and equate them to relevant \(MCs\):

\[
\begin{align*}
20 - 0.2Q_X - 0.05Q_Y &= 1 + 0.1Q_X \\
70 - 0.6Q_Y - 0.1Q_X &= 2 + 0.25Q_Y
\end{align*}
\]

which yields

\[
\begin{align*}
19 &= 0.3Q_X + 0.05Q_Y \\
68 &= 0.85Q_Y + 0.1Q_X
\end{align*}
\]

and \(Q_X = 51\) and \(Q_Y = 74\). \(P_X = 20 - 0.1(51) - 0.05(74) = \$11.2\) and \(P_Y = 70 - 0.3(74) - 0.1(51) = \$42.7\). Goods are substitutes since \(\partial Q_X / \partial P_Y > 0\), and \(\partial Q_Y / \partial P_X > 0\).

6. a. First, write inverse demand schedules from demand functions:

\[
\begin{align*}
Q_X &= 200000 - 1000P_X \quad \rightarrow \quad P_X = 200 - 0.001Q_X \\
Q_Y &= 180000 - 2000P_Y \quad \rightarrow \quad P_Y = 90 - 0.0005Q_Y
\end{align*}
\]

Then find \(MR\) from joint sales,

\[
MR_J = MR_X = 200 - 0.002Q_X + MR_Y = 90 - 0.001Q_Y = 290 + 0.003Q.
\]

And, from \(MR = MC\),

\[
290 + 0.003Q = 50 + 0.001Q \quad \rightarrow \quad Q^* = 60000 \text{ drums}.
\]
\[ P_X = 200 - 0.001(60000) = $140 \quad \text{and} \quad P_Y = 90 - 0.0005(60000) = $60. \]

b. From \( MR = MC \),
\[ 290 + 0.003Q = 3.3 + 0.00005Q \rightarrow Q^* = 94000 \text{ drums}. \]
Check that \( MRs \geq MC \) with this amount:
\[ MR_X = 200 - 0.002(94000) = 12 \geq MC = 3.3 + 0.00005(94000) = $8, \quad \text{and} \]
However, since \( MR_Y = 90 - 0.001(94000) = -4 < 0 \), we have to reconsider the production. First, maximize profit from \( X \) and then maximize revenue from \( Y \):
\[ MR_X = 200 - 0.002Q \quad \rightarrow \quad Q^*_X = 95951 \text{ drums} \]
\[ MR_Y = 90 - 0.0005Q \quad \rightarrow \quad Q^*_Y = 90000 \text{ drums} \]
and \( P_X = 200 - 0.001(95051) = $104 \quad \text{and} \quad P_Y = 90 - 0.0005(90000) = $45. \)

7. First, we have to find the leader’s demand schedule from the difference between market demand and the supply function of the followers:
\[
\begin{align*}
\text{Market demand} & \quad Q_T = (20000/4) - (1/4)P, \quad \rightarrow \quad Q_T = 5000 - 0.25P \\
\text{Followers’ supply} & \quad Q_F = (MC/4) - (2000/4) \quad \rightarrow \quad Q_F = -500 + 0.25P \\
\text{Leader’s demand} & \quad Q_L = Q_T - Q_F \quad \rightarrow \quad Q_L = 5500 - 0.5P
\end{align*}
\]

Now, rewriting \( Q_L \) in terms of \( P_L = 11000 - 2Q_L \), we can find the optimal output produced by leader and its price:
\[ MR_L = 11000 - 4Q_L \quad \rightarrow \quad Q_L^* = 666.67 \quad \rightarrow \quad P_L = $9666.67 \]

b. \( Q_T = 5000 - 0.25(9666.67) = 2583.33, Q_F = 1916.67. \)

8. a. Oligopoly.

b. \[ MR = \begin{cases} 
150 - Q \quad & \text{when} \; 0 \leq Q \leq 50, \\
200 - 3Q \quad & \text{for} \; Q > 50
\end{cases} \]
\[ MC = 15 + Q. \]
When \( Q = 50, \; $50 \leq MR \leq $100 \; \text{and} \; MC = $75 \), so at the kinked \( (Q = 50) \), \( MC \) crosses \( MR \) in the discontinues part. Thus, \( P^* = $125 \) and \( Q^* = 50. \)

c. \[
\begin{array}{c|c|c|c|c}
MC & MC(Q = 50) & \text{Solve} & P^* & Q^* \\
\hline
45 + Q & 95 & - & $125 & 50 \\
15 + 2Q & 135 & 150 - Q = 15 + 2Q & $127.5 & 45 \\
5 + 0.5Q & 30 & 200 - 3Q = 5 + 0.5Q & $116.43 & 55.71 \\
\end{array}
\]
If $MC$ is between $50$ and $100$ when $Q = 50$, we don’t need to solve for optimal price and output. The optimal solution will be at the kinked.

9.

U.S.: $OMC = -1/(1 + (-2)) = 1$ so, $P = $6(1 + 1) = $12.

Overseas: $OMC = -1/(1 + (-3)) = 0.5$ so, $P = $4.5(1 + 0.5) = $6.75.

10. a. First using ATC write $TC = 9Q + 0.11Q^2$ and $MC = 9 + 0.22Q$. Now, equate $MR_1 = MC$ and $MR_2 = MC$ to find optimal $Q$s under price discrimination. From $P_1 = (170/1.9) - (1/1.90)Q_1$, derive $MR_1 = (170/1.9) - (1/1.90)Q_1$ and find that $Q_1^* = 63.234$. Similarly, from $P_2 = (65/0.45) - (1/0.45)Q_2$, derive $MR_2 = (65/0.45) - (2/0.45)Q_2$ and find that $Q_2^* = 29.0376$. Thus, $P_1^* = (170/1.9) - (1/1.90)63.234 = $134.82, and $P_2 = (65/0.45) - (1/0.45)29.0376 = $79.92.

b. First derive the overall demand by summing $Q_1$ and $Q_2$ as $Q = Q_1 + Q_2 = 235 - 2.35P$ and rewrite the inverse demand: $P = 100 - (2/2.35)Q$. Then equate $MR = 100 - (2/2.35)Q = MC = 9 + 0.22Q$ and find that $Q^* = 84.962$ and $P^* = $63.846.

c. Price Discrimination $\pi = $9079

No-Price Discrimination $\pi = $3866.