Exercise for Final Examination

1. A lemon-growing cartel consists of four orchards. Their total cost functions are

\[ TC_1 = 20 + 5Q_1^2 \]
\[ TC_2 = 25 + 3Q_2^2 \]
\[ TC_3 = 15 + 4Q_3^2 \]
\[ TC_4 = 20 + 6Q_4^2 \]

where \( TC \) is in hundreds of dollars, \( Q \) is in cartons per month picked and shipped.

a. Tabulate total, average and marginal costs for each firm for output levels between 1 and 5 cartons per month.

b. If the cartel decided to ship 10 cartons per month and set price of $25 per carton, how should output be allocated among the firms.

c. At this shipping level, which firms has the most incentive to cheat? Does any firm not have an incentive to cheat?

2. Suppose that two identical firms produce widgets and they are the only firms in the market. Their costs are given by \( TC_i = 30Q_i \) \( i = 1, 2 \) where \( Q_i \) is the output of firm \( i \). The demand for the widgets is \( P = 150 - Q \) where \( Q = Q_1 + Q_2 \).

a. Find the Cournot-Nash (noncooperative) equilibrium and calculate the profit of each firm.

b. Suppose the two firms form a cartel. How many widgets will be produced? Calculate the profit of each firm.

c. Suppose firm 1 is the only producer in the industry. How would the market output, and firm 1’s profit differ from that found in part (b).

3. The House Product Division the firm manufactures and sells digital clock radios. A major component of these are supplied by the electronic division. The cost function for electronic division and the radio are respectively,

\[ TC_e = 70 + 6Q_e + Q_e^2 \]
\[ TC_r = 30 + 2Q_r \]

where \( TC_r \) does not include the cost of the component. Manufacture of one radio set requires the use of one electronic component. Market studies show that the firm’s demand curve for the digital radio is given by

\[ P_r = 108 - Q_r \]

a. Assuming no outside market for the components, how many of them should be produced to maximize profits of the firm as a whole? What is the optimal transfer price?

b. If other firms are willing to purchase in the outside market the component manufactured by the electronic division, what is the optimal transfer price?
What price should be charged in the outside market? How many units will the electronic division supply internally and outside market? (Note: The demand for the component in the outside market is \( P_e = 72 - 1.5Q_e \).

4. Consider a firm that produces using two plants, \( A \) and \( B \), with the following marginal cost functions:

\[
MC_A = 10 + 0.01Q_A \\
MC_B = 4 + 0.03Q_B
\]

Find optimal amount produced in each plant and at what price?

5. A firm sells two goods\((X \text{ and } Y)\) that are related in consumption. The estimated demand and cost conditions are:

\[
\begin{align*}
P_X &= 20 - 0.1Q_X - 0.05Q_Y \\
P_Y &= 70 - 0.3Q_Y - 0.1Q_X \\
MC_X &= 1 + 0.1Q_X \\
MC_Y &= 2 + 0.25Q_Y
\end{align*}
\]

Calculate profit maximizing levels of output and prices for the two goods? Are these goods complement or substitutes?

6. ChemTech Corporation produces refined chemicals, and two of these chemicals are complements in production. As it refines the raw chemical input, the processes yield equal amounts of xylene and ylene, denoted as \( X \) and \( Y \) respectively (In other words, one drum of raw chemical input yields one drum of xylene and one drum of ylene). The manager of ChemTech must determine the profit maximizing amounts of xylene and ylene to produce and the prices to charge. The manager has forecasts of the demand functions for the two products:

\[
\begin{align*}
Q_X &= 200000 - 1000P_X \\
Q_Y &= 180000 - 2000P_Y
\end{align*}
\]

where quantities are measured in 55 gallon drums and prices are in dollars per drum. The marginal cost function for refining the raw chemical input is estimated to be \( MC = 50 + 0.001Q \) where \( Q \) is the number of drums of joint product, \( Q_X + Q_Y \).

a. What is the profit maximizing level of production for the joint product? What price should the manager charge for xylene and ylene in order to maximize profit?

b. A technological innovation in chemical processing reduces the marginal cost of production to \( MC = 3.3 + 0.00005Q \). What is the profit maximizing level of production for the joint product? What price should the manager charge for xylene and ylene in order to maximize profit?

7. Alchem (\( L \)) is the price leader in the polyglue market. All 10 other manufacturers (follower (\( F \)) firms) sell polyglue at the same price as Alchem. Alchem allows the other firms to sell as much as they wish at the established price and
supplies the remainder of the demand itself. Total demand for polyglue is given by the following function

\[ P = 20000 - 4Q_T \quad \text{where} \quad Q_T = Q_L + Q_F \]

Alchem’s marginal cost for manufacturing and selling is \( MC_L = 5000 + 5Q_L \). The aggregate marginal cost function for the other manufacturers of polyglue is

\[ \sum MC_F = 2000 + 4Q_F \]

a. To maximize profits, how much polyglue should Alchem produce and what price should it charge?

b. What is the total market demand for polyglue at the price established by Alchem in part (a)? How much of total demand do the follower firms supply?

8. Chillman Motors Inc. believes it faces the following segmented demand function:

\[ P = \begin{cases} 
150 - 0.5Q & \text{when } 0 \leq Q \leq 50, \\
200 - 1.5Q & \text{for } Q > 50
\end{cases} \]

a. What type of industry structure is indicated by this relationship?

b. Calculate the marginal revenue function facing Chillman. When Chillman’s cost is \( TC = 500 + 15Q + 0.5Q^2 \), calculate the Chillman’s profit maximizing price and output combination?

c. What is Chillman’s profit-maximizing price and output combination if total costs change to

\[ \begin{align*}
TC &= 500 + 45Q + 0.5Q^2, \quad \text{or} \\
TC &= 500 + 15Q + Q^2, \quad \text{or} \\
TC &= 500 + 5Q + 0.25Q^2
\end{align*} \]

9. The price elasticity of demand for a textbook sold in the United States is estimated to be -2.0 whereas the price elasticity of demand for books sold overseas is -3.0. The U.S. market requires hardcover books with a marginal cost of $6; the overseas market is normally served with softcover texts, having marginal cost of only $4.5. Calculate the profit maximizing price in each market.

10. JVA Foodstuffs cans green beans for two distinct markets. JVA estimates the demand in each market to be

\[ \begin{align*}
\text{Market 1:} & \quad Q_1 = 170 - 1.90P_1 \\
\text{Market 2:} & \quad Q_2 = 65 - 0.45P_2
\end{align*} \]

The average total cost of producing the green beans is \( ATC = 9 + 0.11Q \).

a. If the markets are such that price discrimination is possible, what will be the price charged in each market?

b. If the markets are such that price discrimination cannot be practiced, what will be the price charged in each market?

c. Would the firm earn higher profits with or without price discrimination?