Lending under Limited Enforcement and Export-Led Growth

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Abstract

This paper develops a model of a two-sector small open economy with locked-in capital accumulation and limited enforcement of foreign debt. Two possible self-enforcing lending schemes are studied. When lending is incentive constrained, a debtor’s repayment utility never falls below its default option. Alternatively, creditors may set target debt/export ratios to keep repayment resources at least as large as default at every point in time. Simulations of the model by genetic algorithms under a wide range of parameter sets demonstrate that the two lending policies generate identical equilibria. We analytically solve the repayment problem under the target debt/export constraint for its long-run equilibrium, and explicitly show the extent of inefficiencies due to limited enforcement. An irreversible export-led growth strategy though mitigating the commitment problem inherent in sovereign lending, can not support full enforcement openness.

Key Words: Limited Enforcement, Irreversible Investment, Export-Led Growth

JEL Classification: F34, F43, F21

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1 Introduction

It is widely recognized that the well established principle of sovereign immunity poses an additional risk for the lenders of international capital. The lack of any notable collateral and the absence of a supranational institution with a recognized authority to enforce loan contracts limit lending to sovereign states. Assuming participants in international capital markets are rational, any sovereign loan contract must therefore be self-enforcing.

A number of studies have indicated that credible sanctions against arbitrary defaults, implicit in the nature of the debtor/creditor relationship, can explain positive sovereign debt\(^1\). In an influential paper, Eaton and Gersovitz (1981) focus on the threat of exclusion of a defaulter from further borrowing. In their model, foreign borrowing cushions consumption from shocks to income so that a greater variability in export earnings increases the option value of access to credit markets thus diminishing the risk of a default. Bulow and Rogoff (1989) show, however, that if a defaulter can enter into outside financial agreements, the threat of exclusion from future borrowing alone will not support foreign lending. In a more recent study, Kletzer and Wright (1998) demonstrate that in a repeated borrowing/repayment game, if the defaulters of today can become defaultees of the future, then positive lending can be sustained without recourse to any exogenous punishments even with outside financial options. In general, incentive constrained consumption loans are inefficient since they can not provide for complete consumption smoothing.

For a developing economy foreign borrowing is desirable not only because it enables consumption smoothing, but more significantly, because it can free investment from the domestic savings constraint. For growth, the level of exports matters as well: exports facilitate imports of investment goods for which domestic substitutes usually do not exist. Hence, the prospect of losing valuable export markets can also work as a strong deterrent against an arbitrary default. Consequently, the extent of a debtor’s dependence on creditor markets for exports becomes strategic. While building up an export presence in creditor markets is a signal of future willingness to repay debt, an expansion in the home goods market will be construed as a sign of potential recalcitrance. Thus, investment and lending decisions are strategi-

cally intertwined distorting incentives for investment and creating room for inefficiencies to arise.

In a two-period stochastic model, Aizenman (1988), for instance, assumes default sanctions in the form of a terms-of-trade deterioration and shows that in the presence of default risk and limited enforcement, investment subsidies to open sectors may be optimal. Nonetheless, investments in the open sectors are lower than full commitment levels resulting in inefficiently small openness\(^2\).

The two-period assumption is unnecessarily restrictive as dynamic adjustment paths are ruled out by construction. In a full fledged intertemporal setting, incentive constrained optimization problems can, however, quickly become analytically intractable. As is usually the case, one must resort to numerical simulations which themselves are complicated by the presence of additional incentive constraints\(^3\). Borenzstein and Ghosh (1989), adopting similar default sanctions as in Aizenman, use dynamic programming methods to numerically optimize a two-sector infinite horizon economy under incentive constraints in a deterministic setting. Though their numerical results confirm the intuition in Aizenman’s two-period model, they fall short of quantifying the extent of likely efficiency loss due to limited enforcement. Dynamic programming techniques are computationally intensive and not particularly well suited for comparative dynamic exercises in large state spaces.

A more serious drawback in the above studies is that the assumed form of sanctions, a terms-of-trade deterioration, lack credibility without some stickiness in capital stocks. A defaulter, anticipating an imminent worsening in its term of trade, can rapidly divest the export capital and use the proceeds for consumption and investment in the import-competing sector, and thereby avoid any default costs. In models with limited enforcement, some irreversibility in capital stocks is essential for the credibility of sanctions. As such, we believe it is one of the key identifying principles in foreign lending

\(^2\)Diwan (1990), on the other hand, stipulating a complete isolation of a defaulter from international trade, finds that a protracted debt renegotiation and a large inherited debt will tilt the \(ex-post\) investment incentives towards the import-competing sector. Sachs (1985), Corden (1989), Helpman (1989), also focus on the adverse effect of heavily discounted inherited debt on the debtor’s incentive to pursue aggressive growth strategies. However, possible adverse incentive effects of a reduction in the debt overhang on creditor countries to keep their markets open to debtor country exports are underplayed.

\(^3\)Marcet and Marimon (1994) Benhabib and Rustichini(1996) study one-sector growth paths with limited enforcement.
for export oriented investment projects.

Admittedly, the full weight of the irreversibility assumption would be felt more in a general stochastic environment where, given an adverse shock of sufficient magnitude, a default will be optimum in equilibrium. In an uncertain world with irreversible investments, a debtor is best advised to keep its default option open. That is, in order to avoid inordinate potential default costs, less openness will be desirable. In a world of certainty such as ours, irreversibility is still needed for, otherwise, the credibility of the lending equilibrium under the threat of trade sanctions is questionable.

The literature on sovereign debt has shown that in the absence of an outside authority to enforce loan agreements, endogenous commitment mechanisms are likely to arise to mitigate, but not fully reverse, the welfare loss due to the lack of enforcement. This paper focuses on one such commitment mechanism, export-led growth, and also quantifies the welfare loss stemming from limited enforcement. We develop a model of a two-sector open economy that underscores the strategic nature of investment activities and foreign lending. Different from its predecessors, however, exports are not domestically consumed, and sector specific capital stocks are locked—so that the threat of trade restrictions is credible. In the absence of full enforcement, the model underlines the commitment value of investment activities and shows how export-led growth can be supported as a second-best policy.

We consider two lending schemes that creditors may look to in order to cope with a potential default. The first is to design a loan contract that secures the debtor repayment utilities to be not less than default at every point in time. By definition, the loan plan is incentive compatible, thus self-enforced. The second is to ensure that the debtor’s repayment resources are no less than its default option at any point in time. We show that the second strategy boils down to setting target debt/export ratios. As the debtor’s utility functional does not change from a repayment to a default, the latter scheme is self-enforced as well.

The repayment program can not be solved analytically under dynamic incentive constraints. Instead, we develop a novel numerical solution method based on genetic algorithms to approximate the model. Since genetic algorithms are parsimonious users of computer memory, they are relatively free from the state space limitations of dynamic programming techniques. We simulate the incentive constrained repayment program under various parameter configurations.

Next, using the same set of parameters, we again use genetic algorithms
to numerically optimize the repayment program now constrained by target debt/export ratios. We show that target debt/export ratios accurately approximate incentive constrained lending. This conclusion is interesting in view of the widespread use of debt/export ratios as measures of creditworthiness by international credit institutions.

Finally, using target debt/export ratios, we are also able to derive analytical results that explicitly show the nature and the extent of the inefficiencies caused by limited enforcement. For instance, if a borrower’s pure time preference exceeds the world interest rate, then the long-run productive capacities in the export and the home goods sectors depend also on the severity of creditor sanctions. More specifically, the marginal product of capital in the export sector is now bounded from above by the domestic pure time preference, as in autarky (no enforcement) and from below by the world interest rate, as in full integration (full enforcement). The size of the home goods, on the other hand, depends both on preferences and the available sanctions. If the home and the foreign goods are gross substitutes, then the home goods sector expands with limited sanctions.

When the pure time preference and the world interest rate are equal, so too is the marginal product of export capital in the long-run, regardless of how limited the sanctions are. The size of the home goods sector, however, is still inversely related to the potential sanctions. Ultimately, to the degree the home goods are poor substitutes for foreign goods, the future consumption is discounted heavily, and the potential sanctions are low, the welfare cost of limited enforcement will be substantial.

The balance of the paper is organized as follows: In the following section we discuss the importance of the relational view of sovereign lending. The model, its justification, and simulation results are provided in section (3). In section (4), we discuss the long-run analytical results. Section (5) concludes.

2 A relational view of sovereign debt

In contrast to the traditional view that the central difference between domestic and international economic relations is that factors of production are mobile domestically and immobile internationally, we believe the main difference between domestic and international economic relations is in how markets function, and that this difference is directly attributable to the sovereign nature of nation states. Domestic economic relations usually take place within
a relatively well-defined set of rules. International economic relations, however, cross national boundaries and involve different legal and institutional systems. If sovereign entities are parties to a transaction, as is the case in much of international lending to developing countries, the relationships are not encompassed by an exogenous institutional and legal structure\(^4\). The relationship between sovereign entities itself generates the “international order” that controls the nature of agreements it is useful to enter into. Limited international capital mobility is just one manifestation of the lack of legal and institutional support for international economic relations.

In a sense, our argument for the importance of openness for development is the reverse of the view that markets must be international in scope to be efficient\(^5\). We argue that international exchange entails higher transactions costs than domestic exchange due to the absence of the normal institutional and legal structures to meet the contingency needs of international market transactions. An institutional and legal framework to support contracts is particularly important in transactions that unfold over time such as long-term development lending. The absence of conventional institutional and legal structures to support international lending provides incentive for the development of alternative mechanisms to support long-term international debtor/creditor relationships.

We emphasize that one way of supporting a long-term debtor/creditor relationship is by establishing a broad long-term relationship, an integral part of which is the debtor/creditor relationship. The value of the relationship in its entirety collateralizes the debt and assures that borrowers will make strong efforts to repay even when any further borrowing will most likely be limited.

\(^4\)For an interesting discussion of this point, see Kindleberger (1970, p 37).

\(^5\)Numerous empirical studies have found that developing countries that have adopted outward-oriented growth strategies have achieved higher growth rates than countries that have pursued inward-looking growth policies. See, for instance, Kavoussi (1984), Corbo, Krueger, and Ossa (1985), Barro (1991), Levine and Renelt (1992), Levin and Raut (1997). Explanations for the relative success of outward-oriented trade policies have focused on economies of scale in production and the role of international markets in providing economic and political discipline. It has been argued that domestic markets are too small for efficient levels of production for many industrial goods, and that production for protected domestic markets breeds sloth, inefficiency, and encourages rent seeking behavior. Others Edwards (1989, 1992) and Grossman and Helpman (1990) have focused on the benefits of openness as increased contact with the more advanced economies speeds up knowledge spillovers and absorption which in turn accelerate growth in the developing world.
Scale effects in international economic relations result from the proportion between the long-term commitment of a country to international relations and the scale of international debt the country can believably commit itself willing to repay. Piecemeal or casual international economic interaction will not collateralize large international debts. In order to engage in large scale economic transactions, a country must make a commitment of sufficient magnitude to provide assurance that it will not find it in its interest to engage in the sort of short-run maximizing (opportunistic) behavior that would be the outcome of maximizing the gain from one time or occasional or casual economic interactions. This is the reason for the distinction, important for international financial relations, between the ability to repay and the willingness to repay.

The degree of integration of a debtor to a trading system is a crucial element in a default decision since repudiation of international debt may be accompanied by a rupture of relations between a debtor and its trading block. The costs suffered by a defaulter depend on the vulnerability of the debtor to the creditor sanctions. A higher degree of integration will bring about a higher degree of openness, and thus will raise the debtor vulnerability to creditor sanctions. What is important, is not the degree of integration when lending starts, but rather the extent of integration that will have been achieved, due in part to credit flows, when lending ceases.

The depth of integration itself is a complex measure. It encompasses security, political, cultural, and economic dimensions. We focus upon economic factors that measure the degree of commitment to the integration process. The sectoral specificity and the durable nature of capital make the pattern of capital accumulation a good candidate for signaling a credible commitment. This draws attention to the strategic nature of investment activities and growth policies of a potential borrower.

Commitment to integration needs to be feasible from a political economy point of view as well. Debt financed export-led growth creates constituencies in creditor and debtor countries for trade and debt repayment. In addition to consumer interests, the financial sector in the creditor country and the export sector in the debtor country, will be a political constituency for trade and debt repayment. Debt repudiation would be harmful to the interests of the trade coalition while trade restrictions in the creditor country would hurt the interests of the constituency for debt repayment in the debtor country. Therefore, the threats supporting lending for export-led growth will have
credibility.

3 Model

Consider a small open economy that produces an export, \( Q_e = Q_e(K_e) \), and a home good, \( Q_h = Q_h(K_h) \). The production technology in each sector is strictly concave. Home goods are only consumed so that \( C_h = Q_h(K_h) \) in equilibrium where \( C_h \) denotes the consumption of home goods.

Sector-specific capital stocks, \( K_e \) and \( K_h \), grow over time according to,

\[
\dot{K}_e = J_e, \quad \text{and} \quad J_e \geq 0, \quad (1)
\]
\[
\dot{K}_h = J_h, \quad \text{and} \quad J_h \geq 0. \quad (2)
\]

Time is continuous, and unless otherwise stated, all variables are functions of time. A dot over a variable denotes its time-derivative, and \( J_e \) and \( J_h \) are the amounts of imported goods invested in the respective sectors. The absence of domestic production, irreversibility and sectoral specificity of investment are adopted to highlight the value of investment as a commitment mechanism in the strategic interplay between debtors and creditors in an environment of limited enforcement.

Current account deficits can be financed by foreign borrowing so long as the outstanding debt is below the credit limit,

\[
\dot{D} = rD + C_m + J_e + \frac{z}{2} J^2_e + J_h + \frac{z}{2} J^2_h - Q_e,
\]

\[
D \leq \bar{D}, \quad (3)
\]

where \( D \) is the outstanding debt, \( \dot{D} \) shows the rate of gross borrowing, and \( \bar{D} \) indicates the credit limit yet to be determined by creditors. Note that the terms of trade is fixed at unity so that borrowing is in terms of foreign goods. Import of consumption goods is \( C_m \). The fixed parameters \( r \) and \( z \) denote the world interest rate and the common investment adjustment coefficient respectively. In each sector, investment involves quadratic adjustment costs and accumulated stocks do not depreciate.

The threat of forfeiture of collateral has two effects: the potential cost to the borrower of losing the collateral and the prospective value to the creditor of gaining the collateral. An asymmetry is required in a relationship to collateralize debt in that a prospective disruption of the relationship must be costly to the debtor, to provide incentive to avoid the sanction, but not costly to the creditor, so the threat of the sanction will be believable.
At any point in time, given the initial stocks, $K_{et}$, $K_{ht}$ and $D_t$, if the debt is repaid, the debtor will then obtain a repayment utility,

$$V^P(K_{et}, K_{ht}, D_t, t; \beta, \alpha) = \max_{C_m, J_e, J_h} \int_t^\infty e^{-\beta(\tau-t)} U(C_m, Q_h) d\tau$$

(4)

where $U(C_m, Q_h)$ is a strictly concave instantaneous utility function, and $\beta$ stands for the fixed pure time preference rate.

Or, the debtor may opt for a default with the certain prospect of collective creditor sanctions of an indefinite duration: cutting off of lending and retaliatory measures which will disturb the debtor’s normal flow of trade\(^7\). A defaulter has to conduct trade on a current basis, $\dot{D} = 0$, and will also come under direct trade sanctions. The external constraint, equation (3), then becomes,

$$C_m + J_e + \frac{z}{2} J^2_e + J_h + \frac{z}{2} J^2_h = (1 - \alpha) Q_e \text{.}$$

(5)

This parameterization of sanctions is in keeping with the literature on sovereign debt\(^8\). The “sanction or enforcement coefficient”, $0 \leq \alpha \leq 1$, can be motivated by arguments on various levels. For example, if a willful default is countered by tariffs on the defaulter’s exports in creditor countries, then $\alpha$ is the percentage deterioration in the defaulter’s terms of trade so that $\alpha Q_e(K_{et})$ is the direct default cost flow. Alternatively, if quantity restrictions are in force, then $\alpha$ is the fraction by which exports are reduced implying that the potential default cost flow is the same, namely, $\alpha Q_e(K_{et})$. At a more general level, $\alpha$ measures the difficulty with which the debtor can shift its trade to alternative markets at favorable terms, and also the severity of trade restrictions it anticipates facing in creditor markets. As such, it parametrizes the dependency of the debtor on the creditor markets for exports and thus the ability of creditors to enforce sanctions.

If the debtor defaults, it will obtain a default utility,

$$V^D(K_{et}, K_{ht}, t; \beta, \alpha) = \max_{C_m, J_e, J_h} \int_t^\infty e^{-\beta(\tau-t)} U(C_m, Q_h) d\tau$$

(6)

\(^7\)In a more general stochastic environment, the distinction between a default due to circumstances outside the control of the debtor and an outright repudiation would become important in the design of loan contracts. In our deterministic setup we focus on the effects of limited enforcement on growth.

\(^8\)For similar specifications see for instance, Sachs(1985), Cohen and Sachs(1986). Also, in contrast with Diwan(1990), the trade loss is partial here.
with the same restraints binding except now equation (5) replaces (3).

Having set up the debtor’s problem, we now turn to the determination of the credit ceiling, $\tilde{D}$ by the creditors.

3.1 Lending

Consider two lending scenarios. In the first, the loan contract is dynamically incentive constrained. That is, given any $K_{et}$ and $K_{ht}$, the contract guarantees that the debtor’s repayment utility will be no less than its default option at any point in time. In the second, creditors adopt a lending rule that ensures that the debtor’s repayment resources never fall below the default alternative at any point in time.

3.1.1 Incentive constrained lending

Note that a rational, self-interested sovereign debtor will default if and when $V^P(K_{et}, K_{ht}, D_t, t; \beta, r) < V^R(K_{et}, K_{ht}, t; \beta, \alpha)$. Thus, creditors’ loan plan must induce such debt and growth paths, $D_t$, $K_{et}$, and $K_{ht}$ that assures $V^P(K_{et}, K_{ht}, D_t, t; \beta, r) \geq V^R(K_{et}, K_{ht}, t; \beta, \alpha)$ at every point in time. Because of the concavity of production and instantaneous utility functions, the repayment utility, $V^P(K_{et}, K_{ht}, D_t, t; \beta, r)$, is decreasing in $D_t$. Hence, the maximum sustainable credit, $\tilde{D}_t$, solves the functional equation, $V^P(K_{et}, K_{ht}, \tilde{D}_t, t; \beta, r) = V^R(K_{et}, K_{ht}, t; \beta, \alpha)$ for all time. Given the nonlinearities in our model, an analytical solution to the optimal credit limit, $\tilde{D}_t$, is not available.

In essence, creditors seek a lending policy that maximizes the debtor’s repayment utility while satisfying the additional incentive constraint, $V^P(K_{et}, K_{ht}, D_t, t; \beta, r) \geq V^R(K_{et}, K_{ht}, t; \beta, \alpha)$. The problem is in the general class of dynamic incentive constrained optimal control problems. Marcet and Marimon (1994) and Rustichini (1998a) propose use of lagrange multiplier techniques to solve similar problems. For numerical approximation, dynamic programming methods are suggested by Rustichini (1998b). However, the proposed methods are computationally inefficient for problems with large state spaces such as ours.

Our solution algorithm is also a contribution to the computational methods in this area. We develop a numerical routine which uses genetic algorithms to recursively eliminate probable defection (default) states so as to construct the equilibrium paths. Essentially, since there can be no default
in equilibrium, the feasible equilibrium states, $K_{ct}, K_{ht}$ and $D_t$, must all satisfy the incentive constraint and must all be accessible through repayment. The algorithm uses a Darwinian evolutionary search to find and improve upon feasible equilibrium states in order to maximize the repayment utility. The details of the algorithm and the numerical parameters are provided in Appendix A.

Since the purpose of our numerical experimentation is to discover how limited enforcement impacts equilibrium paths, we approximate the repayment and default programs with a low, $\alpha = 0.1$, and a high enforcement, $\alpha = 0.8$, rates. The equilibrium paths are also affected by the relative magnitudes of the pure time preference and the interest rates so that the experiments are run with $\beta = r$ and $\beta > r$. Altogether, four cases are considered$^9$.

In order to control for the effects of technology and tastes on the experiments, we assume symmetry in production technologies and preferences over domestic and foreign goods. The economy is assumed initially to be almost closed with no outstanding foreign liabilities and a relatively small export sector. The numerical findings are summarized in Figures and Tables 1 and 2.

As shown in Figure 1, the debtor starts with no foreign debt and a relatively low export capacity, and quickly accumulates both. The home goods sector expands as well, albeit, at a slower pace. If $\beta > r$ and $\alpha < 1$, incentive constraints eventually bind and remain tight thereafter. That is, for some $t$, $D_s = \tilde{D}_s$ for all $s \geq t$.

Increased enforcement speeds up the adjustment process and allows debtor to shift consumption to earlier periods. Since, the home goods are only consumed, the rapid initial growth in the home goods sector indicates the borrower’s impatience to attain the long-run consumption level. Furthermore, since the home goods sector shrinks with a stronger enforcement in the long-run, adjustments run their course faster.

In a tight enforcement regime, larger initial inflows also finance higher consumption of foreign goods while exports are still low. When the debtor’s desire to consume in the present is strong, $\beta > r$, and severe sanctions are in force against a default, $\alpha = 0.8$, the time profile of foreign goods consumption starts to decline. If, $\beta = r$, as in Figure 2, consumption shifting is less desired so that time profiles become more balanced and increase monotonically. In

$^9$Since we are limited by computational costs, we can not experiment with other parameters of the model.
Figure 1: Consumption and capital under incentive constraints ($\beta > r$)

this instance, earlier capital inflows largely finance investment in both sectors.

Ultimately, a tighter enforcement, as captured by a higher $\alpha$, reduces the effective opportunity cost of foreign borrowing. The borrower not only invests and consumes more in the present, but also invests relatively more in the export sector and consumes more of the imported goods. To the extent home goods can substitute for foreign goods with ease, as assumed in our exercises, the loss of welfare due to the lack of enforcement is limited. Otherwise, with a wide margin between the domestic time preference and the world interest rate, the welfare cost can be substantial.
3.1.2 Target debt/export ratios

Consider now the alternate lending scenario wherein lenders recognize the inherent time-inconsistency in extending loans to sovereign countries, but lack the capacity to compute the dynamically incentive compatible loan contracts. Neither they nor the debtor deem the pledges of future investment or loan plans as credible. Furthermore, the creditors are unable to compute the debtor’s optimal default policies. Instead, loans are collectively made to assure that the debtor’s interest burden at any time not become too onerous, creating temptation for a default.

Creditors reason that by defaulting the debtor would save the face value of its outstanding debt, $D_t$, and given its locked-in export capacity, it would at the minimum suffer a default penalty equivalent to the capitalized value of the current default cost, $\alpha Q_e(K_{et})/r$. They reckon that if the outstanding debt is not allowed to exceed the minimum default penalty, $D_t \leq \alpha Q_e(K_{et})/r$. 

Figure 2: Consumption and capital under incentive constraints ($\beta = r$)
Table 1: Comparison of Lending Strategies, $\beta > r$

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<th>$V^R$</th>
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NOTE: In this experiment, $\beta = 0.09$, $r = 0.08$, and all other parameter values and functional forms are discussed in Appendix A.

then a potential default can be averted. Below, Proposition 1 establishes that this lending policy is self-enforcing.

**Proposition 1:** The lending policy that sets a target debt to export ratio, $\tilde{D}_t = \alpha Q_e(K_{et})/r$, is self-enforcing. Thus, creditors can prevent a default if they insist that the debt to export ratio at any given time, $D_t/Q_{et}$, not exceed a “vulnerability ratio”, $\alpha/r$.

**Proof.** Assume $\beta \geq r$ so that borrowing is desirable, $\tilde{D}_t \geq 0$. Note that the
Table 2: Comparison of Lending Strategies, $\beta = r$

<table>
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<th>$\alpha$</th>
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NOTE: In this experiment, $\beta = r = 0.09$.

repayment and default programs are essentially the same except for the external constraints, (3) and (5). Indeed, if $\tilde{D}_t - r\tilde{D}_t + \alpha Q_e (K_{et}) = 0$, for all $t$, then $V^P(K_{et}, K_{ht}, D_t, t; \beta, r) = V^R(K_{et}, K_{ht}, t; \beta, \alpha)$ for all $t$ because both programs have identical utility functionals and equal resources. Suppose $rD_t - \tilde{D}_t \leq \alpha Q_e (K_{et})$ for all $t$. Then, $V^P(K_{et}, K_{ht}, D_t, t; \beta, r) \geq V^R(K_{et}, K_{ht}, t; \beta, \alpha)$ for all $t$ as both programs start with the same initial capital stocks, $K_{et}$ and $K_{ht}$, and have identical utility functionals. That is, so long as the net debt service is less than the default cost, the debtor has more resources under the repayment program, and therefore, is better off servicing the debt. Since $\tilde{D}_t \geq 0$, if $rD_t \leq \alpha Q_e (K_{et})$ for all $t$, then $V^P(K_{et}, K_{ht}, D_t, t; \beta, r) \geq V^R(K_{et}, K_{ht}, t; \beta, \alpha)$.
for all \( t \). Thus, Proposition 1 follows.

At first brush, this lending policy may look too restrictive since creditors have ignored new lending \((\dot{D}_t > 0)\) as a repayment resource in their default calculus. In fact, lending will not be unduly restricted. Because, in a world of certainty, a default can be optimal only after all available credits are exhausted. Thus, when weighing the costs and benefits of a default, it is reasonable to assume that credits have been used up.

Our numerical results in the following section also verify Proposition 1. More significantly, when we approximate the repayment program using target debt/export ratios, our numerical results closely replicate those under the dynamic incentive constraints. We speculate that this result is due to the similarity of the repayment and default programs, and not necessarily of a general character in all incentive constrained problems.

### 3.1.3 Simulations with target debt/export ratios

Using the same set of parameters, we first approximate the repayment utilities, \( V^P(K_{et}, K_{ht}, D_t, t; \beta, r) \), now constrained by target debt/export ratios, \( D_t/Q_e \leq \alpha/r \), for comparison with the incentive constrained repayment utilities in the previous section. As observed in Tables 1 and 2, the utilities under the two lending regimes are similar within limits of error tolerance due to approximation. In fact, repayment utilities are slightly better under target debt/export ratios than under incentive constraints. Since the \textit{maximum} self-enforced lending can only be supported under incentive constraints, these small differences in utilities must be due to relatively larger numerical errors in approximating incentive constrained lending. Numerically, incentive constraints are more complex than simple debt/export ratios and thus more difficult for genetic algorithms to learn.

Nonetheless, in order not to leave any room for ambiguity, we verify that the paths generated under target debt/export ratios are self-enforcing. For every time period, we use the capital stocks from the debt/export constrained repayment programs and approximate the default utilities, \( V^R(K_{et}, K_{ht}, t; \beta, \alpha) \). Figures 3 to 5, and Tables 1 and 2 summarize our numerical results. As evidenced, the credit constraint, \((D_t/Q_e) \leq \alpha/r\), guarantees that \( V^R(K_{et}, K_{ht}, t; \beta, \alpha) \leq V^P(K_{et}, K_{ht}, \dot{D}_t, t; \beta, r) \), indicating that the debt paths so generated are indeed self-enforcing.

Notice that in a loose enforcement regime where the borrower is allowed
to hold a debt stock about 1.25 times its exports, $\alpha = 0.1$, the credit limit binds around the tenth quarter (after the first time period) and remains so thereafter. Once the credits tighten, the creditors allow gross lending to grow at the same rate as exports and also start extracting a fixed fraction, $\alpha = 0.1$, of the debtor’s exports as interest payments.

Since, the equilibrium paths under target debt/export ratios closely track the equilibrium paths under incentive constraints, we do not repeat a full discussion of dynamic adjustments here.

In closing, we note that a target debt/export ratio is a credible lending policy as it aligns the debtor’s ex ante and ex post incentives. Ex ante, it imparts to a potential borrower the ability to borrow contingent on its investment and trade policies; ex post, it measures the creditworthiness as the willingness, due in part to the borrower’s past investment decisions, to service a given level of debt.
3.2 Long-run analytical results

Our numerical exercises have shown that target debt/export ratios accurately approximate incentive constrained lending. This conclusion gains more weight in the light of the fact that debt/export ratios have been widely used by international credit institutions as measures of creditworthiness. Despite their widespread practical appeal, however, to our knowledge, an analysis as to the welfare costs of such ratios on growth has been ignored. The following analytical results, therefore, should be of interest both for their empirical relevance, and for the fact that debt/export ratios themselves have a rational foundation as we have demonstrated. We summarize our long-run results as propositions.

Proposition 2: Suppose that creditors adopt the lending policy \( \tilde{D} = (\alpha/r)Q_e(K_e) \). Reconsider the repayment program described by equations (1)
Figure 5: Consumption and capital under target debt/export ratios \( (\beta = r) \) to (4), now constrained by \( D(\tau) \leq (\alpha/r)Q_e(K_e) \).

(i) If \( \beta > r \) and \( 0 \leq \alpha \leq 1 \), then the long-run export capital satisfies:

\[
\frac{\partial Q_e}{\partial K_e^*} = \frac{\beta r}{r(1 - \alpha) + \alpha \beta}
\]

If \( \beta = r \) and \( 0 \leq \alpha \leq 1 \), then the long-run marginal productivity in the export sector is independent of sanctions; namely, \( \partial Q_e/\partial K_e^* = \beta = r \).

(ii) If \( \beta \geq r \), then the long-run capital stock in the home goods sector is determined by both sanctions and preferences:

\[
\frac{\partial U}{\partial C_m^*} = \frac{\partial U}{\partial Q_h(K_h^*)} \frac{\partial Q_h}{\partial K_h^*}.
\]

The proof is in Appendix B. Proposition 2 shows that the long-run marginal productivity in the export sector varies between the world interest and the domestic pure time preference rates, depending on the rate of enforcement.
If no sanctions can be brought to bear upon a debtor, \( \alpha = 0 \), then the financial autarky results in, \( \partial Q_e / \partial K^*_e = \beta \). With full enforcement (\( \alpha = 1 \)), on the other hand, perfect capital market integration leads to a convergence, \( \partial Q_e / \partial K^*_e = r \).

When \( \beta = r \), the long-run productive capacity in the export sector is independent of the severity of sanctions as also witnessed in our simulations. Depending on the initial size of the export sector and \( \alpha \), the credit limit may or may not bind. If the initial export capital and the rate of enforcement are sufficiently large, as per our numerical exercises with \( \alpha = 0.8 \), the credit limit will not bind at all so that no loss of efficiency ensues. Otherwise, (e.g., \( \alpha = 0.1 \)) the credit constraint will eventually bind and remain tight thereafter. Since, in the long-run, all marginal investment opportunities will have been completely exhausted, the value of extra credit will be nil. In this instance, limited enforcement will be costly because, restricted foreign lending will slow down adjustments and will also lead to an inefficiently large home goods sector. As different from the export sector, the size of the home goods sector in the long-run is also affected by the preferences over the two goods.

Note that in a more general stochastic environment a sufficiently large shock will trigger a default and thus sanctions in equilibrium. Consequently, the debtor’s investment plans will have to cover this contingency as well. With default as a real possibility, the long-run sectoral mix as indicated by Proposition 2 would not be optimal. In order to plan for such an eventuality, and not to regret having invested so much in the export sector to bear such heavy default costs, the debtor will invest relatively less in the export sector than prescribed by Proposition 2. In other words, with irreversible investments in an uncertain environment the debtor will always keep its default option open.

Below, Proposition 3 summarizes the sensitivity of the long-run equilibrium to the rate of enforcement as well as making more precise the role preferences play.

**Proposition 3:** If \( \beta > r \), then in the long-run:

\[
\frac{\partial K^*_e}{\partial \alpha} \geq 0, \quad \frac{\partial D^*}{\partial \alpha} > 0, \quad \frac{\partial C^*_m}{\partial \alpha} < 0, \quad \frac{\partial K^*_h}{\partial \alpha} < 0.
\]

If \( \beta = r \), the above comparative static results are still valid, with \( \partial K^*_e / \partial \alpha = 0 \).
The proof is in Appendix B. Proposition 3 establishes that a stronger enforcement will support a larger debt and openness in equilibrium. Since increased inflows also finance foreign consumption and investment in the home goods sector, indebtedness will grow more than exports so that foreign consumption falls in the long-run. Furthermore, as shown in the proof of Proposition 3, if foreign and home goods are gross substitutes, the home goods consumption drops as well. Dynamic inefficiency in the home goods sector occurs in the opposite direction. Namely, lower sanctions lead to an inefficiently large home goods sector. The upshot of the comparative static exercise is that limited enforcement also limits openness. These results are intuitive and also verified by our numerical experiments.

Finally, to conclude our analytical inquiry, and also to better appreciate how the locked-in export capital serves as a commitment mechanism, compare the long-run export capacity when investments are irreversible with the capacity when they could be undone. Had the debtor been able to disinvest after a default, the long-run capital stock in the export sector would have satisfied, $\partial Q_e/\partial K^*_e = \beta/(1 - \alpha)$. Clearly, a defaulter would have preferred to have a smaller export capacity than the one it got locked into. Thus, the capacity created in the export sector under the lending regime, $D_t/Q_e(K_{et}) \leq \alpha/r$, is “too high” should the debtor default. As such, this “excess capacity” provides the guarantee to the creditors that their loans will be repaid.

4 Concluding remarks

In the absence of conventional legal and institutional structures, participants in international loan markets have incentives to develop alternative relational mechanisms to provide support for the contingency needs of loan transactions. Specifically, in an environment of limited enforcement, investment decisions and long-term debtor/creditor relationships become strategically interlocked.

Creditors may set a target debt/export ratio to keep a debtor’s interest burden below a certain fraction of its exports so that servicing debt does not become too onerous triggering a default. Within the repudiation/retaliation framework, this fraction can be construed as a measure of the severity of creditor sanctions that may come about in various guises. Thus, the upper bound on the debt service gains a special meaning as the potential default
cost. We show that, under the assumptions of our model, target debt/export ratios generate self-enforcing trajectories of debt should a default ever be entertained as an option. Moreover, within the parameters of the model, they approximate incentive constrained equilibrium lending closely.

Borrowers, on the other hand, may deliberately alter their investment patterns to increase their vulnerability to potential creditor sanctions to attract increased foreign capital. To the extent investments are irreversible, when present governments commit to export-led growth, they also limit the possibilities for future governments to reoptimize. A large export sector which has been financed by foreign debt is then a “firm pledge” not to default, for the costs of a default have increased as well.

The inefficiencies arising from the lack of enforcement, however, can not completely be done away with. Lending under limited enforcement is too restricted to support an efficient foreign and home goods mix. By and large, the trade sector is relatively too small. To the degree the home goods are poor substitutes for foreign goods, the future is discounted more heavily domestically than by the rest of the world, and the potential sanctions are low, the welfare cost of limited enforcement can be substantial.
Appendix A

Throughout our exercises, we assume identical sectoral production functions, \( Q_i(K_i) = aK_i^b, \ i = e, h \). Consumption preferences are represented by \( U(C_m, C_h) = (C_mC_h)^\gamma \), which is symmetric in the two goods. Two enforcement regimes exist: a low \( \alpha = 0.1 \) and a high \( \alpha = 0.8 \). Other parameter values are\(^{10}\): \( a = 3.57146, \ b = 0.4, \ \gamma = 0.5, \ z = 0.05, \ r = 0.08, \ \beta = 0.09, \ K_e(0) = 30, \ K_h(0) = 50, \) and \( D(0) = 0 \). When \( \beta = r \), the same parameter values are used with the exception, \( \beta = r = 0.09 \).

Assuming both programs become stationary at some arbitrary date \( T \), we discretize the repayment and default programs using the time aggregation method proposed by Mercenier and Michel (1994) to ensure that the discrete models have the same steady-states as their continuous analogs. In time aggregation, we assume 15 periods with a dense equally spaced gridding of the time horizon \( T = 150 \), which is sufficient to capture convergence.

In all of our numerical experiments, we use the genetic operators in the public domain GENESIS package by Grefenstette (1990) on a SUN SPARC-1000 running Solaris 2.5. In a typical run, we use a population size of 50, a crossover rate of 0.60, a mutation rate of 0.03, and a generation number of 200,000. The source codes for the evaluation functions can be accessed at the web site: http://www.bilkent.edu.tr/˜suheyla.

A.1 Genetic algorithms for incentive constrained lending

We first form an initial random sample of candidate solutions (binary strings), \( [K_{et}, K_{ht}, D_t] \) for \( 0 \leq t \leq T \). Assuming no default has occurred until \( T \), if the credit constraint is binding at the steady state so that \( V^R(K_{es}, K_{hs}; \beta, \alpha) = V^P(K_{es}, K_{hs}, D_s; \beta, r) \), for all \( s \geq T \), then both programs must have reached the same stationary capital stocks, \( K_{et} \) and \( K_{ht} \). If the credit constraint is not binding, then \( K_{et} \) and \( K_{ht} \) are not necessarily the same for both programs.

Evaluation procedure runs backward in time. Given a random initial sample, the genetic algorithm starts evaluating the string performance from time \( T \). For a given string, the stationary (as yet suboptimal) repayment, \( V^P \), and the stationary (as yet suboptimal) default, \( V^R \), utilities are computed. The algorithm eliminates the path if it ends with a default. That

\(^{10}\)Similar values are used by Lipton and Sachs (1983) in a similar context, and by Auerbach and Kotlikoff (1987) in a different context.
is, the string is penalized if it ends with stationary states, \((K_{eT}, K_{hT}, \bar{D}_T)\), such that \(V^P(K_{eT}, K_{hT}, \bar{D}_T; \beta, r) < V^R(K_{eT}, K_{hT}; \beta, \alpha)\). Moving one step back in time, and assuming no default has taken before \(T-1\), the algorithm computes the performance of a candidate solution path from states, \((K_{eT-1}, K_{hT-1}, D_{T-1})\), penalizing if the irreversibility is violated and/or 
\(V^P(K_{eT-1}, K_{hT-1}, D_{T-1}, T-1; \beta, r) < V^R(K_{eT-1}, K_{hT-1}, T-1; \beta, \alpha)\). That is, the string is eliminated if the no-default states at time \(T\) are reached by a default at time \(T-1\) for such transitions can not exist in equilibrium. The same procedure is repeated recursively until time zero for all strings in the population.

Next, a mating pool is formed in accordance with the fitness (the performance of a string relative to the population average) of each string. Strings are randomly matched to crossover and with a small probability mutate to form the next generation. We maintain an elitist selection strategy and keep the best performing string intact. The same evaluation procedure above is again employed upon the new generation of candidate solution paths to form fitter generations. Thanks to the evolutionary operators, selection and crossover, better performing strings survive and proliferate until the best solution dominates, and no further improvement can be achieved by recombining the elements of the population. The following is the pseudo code for the algorithm.

\begin{verbatim}
procedure Incentive Constrained GA;
begin
  i=0; /* i is the generation number */
  initialize Ke(i),Kh(i),D(i); /* randomly generated populations */
  t=T;
  evaluate Ke(t),Kh(t),D(t); /* backward string evaluation */
  check VP(t) >= VR(t); /* checking incentive constraints */
  t=t-1;
  repeat until (t=0);
  i = i+1;
  select new population Ke(i),Kh(i) and D(i);
  crossover and mutate Ke(i),Kh(i),D(i);
  t=T;
  evaluate Ke(t),Kh(t),D(t);
  check VP(t) >= VR(t);
  t=t-1;
  repeat until (t=0);

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\end{verbatim}
repeat
    until (terminal condition); /* until a given number of generations*/
end;

where VP(t) and VR(t) are discounted repayment and default utilities respectively.

A.2 Genetic simulations with target debt/export ratios

Using the same parameter configurations, we approximate the repayment program by genetic algorithms. Genetic algorithms are powerful general purpose optimization tools in irregular and complex search spaces. A drawback, however, is the lack of any obvious and generally accepted method of dealing with constraint violations. Given that repayment program is constrained by target debt/export ratios, this difficulty may seem especially troubling. Nonetheless, we successfully incorporate constraint violations into fitness evaluations by way of penalties. The following is a sketch of the algorithm.

procedure Debt/Export GA; begin
    i=0;
    initialize Ke(i),Kh(i),D(i);
    t=0;
    evaluate Ke(t),Kh(t),D(t);
    t=t+1;
    evaluate Ke(t),Kh(t),D(t);
    check (D(t)/Qe(t)) <= "vulnerability ratio"
    repeat until (t=T)
    i=i+1;
    select new population Ke(i),Kh(i) and D(i);
    crossover and mutate Ke(i),Kh(i),D(i);
    t=0;
    evaluate Ke(t),Kh(t),D(t);
    t=t+1;
    evaluate Ke(t),Kh(t),D(t);
    check (D(t)/Qe(t)) <= "vulnerability ratio"
    repeat until (t=T)
    repeat
    until (terminal condition);
end;

where the “vulnerability ratio” is $\alpha/r$. 

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Appendix B

B.1. Proof of Proposition 2. The debtor faces an optimal growth problem with an additional target debt/export constraint. Kamien and Schwartz (1981) provide a set of necessary conditions for optimal control problems with bounded state variables. These involve a new definition of the Hamiltonian, now appended with the total differential of the state variable constraint and an accompanying multiplier which is non-increasing when the constraint is binding and a constant but not necessarily zero during a free interval.

In our problem, \( H \), the current valued Hamiltonian is:

\[
H = U(C_m, Q_h) + \mu [rD + C_m + J_e + \frac{z}{2} J_e^2 + J_h + \frac{z}{2} J_h^2 - Q_e(K_e)] \\
+ \lambda_e J_e + \lambda_h J_h + \phi \{ \alpha (\partial Q_e / \partial K_e) (J_e/r) \\
- [rD + C_m + J_e + \frac{z}{2} J_e^2 + J_h + \frac{z}{2} J_h^2 - Q_e(K_e)] \}.
\]

For optimality, the following first order conditions are necessary:

\[
\frac{\partial U}{\partial C_m} = \phi - \mu, \quad (\text{B.1})
\]

\[
J_e = \frac{1}{z} \left\{ \left[ (\lambda_e + W_1 + \frac{\phi \alpha}{r} \frac{\partial Q_e}{\partial K_e}) / (\phi - \mu) \right] - 1 \right\} \quad (\text{B.2})
\]

\[
J_h = \frac{1}{z} \left\{ \left[ (\lambda_h + W_2) / (\phi - \mu) \right] - 1 \right\} \quad (\text{B.3})
\]

where \( \lambda_e, \lambda_h, \) and \( \mu \) are the respective co-state variables for \( \dot{K}_e, \dot{K}_h, \) and \( \dot{D}. \) The variable, \( \phi \) adjoins the time-differentiated credit constraint, \((\alpha/r)(\partial Q_e/\partial K_e)\dot{K}_e - \dot{D}. \) Optimal policies will also satisfy (1)-(3) as well as \( W_1 \geq 0, J_e W_1 = 0, W_2 \geq 0, J_h W_2 = 0. \)

Equation (B.1) prescribes the optimal consumption of foreign goods while (B.2) and (B.3) indicate the rates of investment in each sector. Note that the term \((\phi \alpha/r)(\partial Q_e/\partial K_e)\) appearing in the numerator in equation (B.2) captures the beneficial effect of investment in the export sector on the credit constraint. Thus, the presence of the credit constraint tilts the optimal composition of investment in favor of the export sector relative to a decentralized borrowing or when the credit constraint is altogether absent.
The variables $\mu$, $\lambda_e$, $\lambda_h$, and $\phi$ evolve according to,

$$
\dot{\mu} = \mu(\beta - r) + \phi r
$$

(B.4)

$$
\dot{\lambda}_e = \lambda_e \beta + (\mu - \phi) \frac{\partial Q_e}{\partial K_e} - (\phi \alpha \frac{J_e}{r}) \frac{\partial^2 Q_e}{\partial K_e^2}
$$

(B.5)

$$
\dot{\lambda}_h = \lambda_h \beta - \frac{\partial U}{\partial C_h}
$$

(B.6)

with the transversality conditions,

$$
\phi \geq 0, \quad (\dot{\phi} - \beta \phi) \leq 0, \quad (\dot{\phi} - \beta \phi)[(\alpha Q_e/r) - D] = 0,
$$

(B.7)

$$
limit_{\tau \to \infty} e^{-\beta(\tau-t)} \mu D_\tau = limit_{\tau \to \infty} e^{-\beta(\tau-t)} \lambda_e K_e = limit_{\tau \to \infty} e^{-\beta(\tau-t)} \lambda_h K_h = 0.
$$

(B.8)

Consider first the export sector. Assuming an interior solution, set $J_e$ in equation (B.2) to zero to obtain,

$$
\lambda_e + \frac{\phi \alpha}{r} \frac{\partial Q_e}{\partial K_e} = \phi - \mu.
$$

(B.9)

Time-differentiate (B.9) and substitute $\dot{\mu}$ and $\dot{\lambda}_e$ from (B.4) and (B.5). Replace $\beta \lambda_e$ in the resultant expression from (B.9) to get,

$$
\frac{\alpha}{r} (\frac{\partial Q_e}{\partial K_e} - r) (\dot{\phi} - \beta \phi) - (\phi - \mu) (\frac{\partial Q_e}{\partial K_e} - r) = 0.
$$

Noting that $\dot{\phi} = \dot{\mu}$, and after a few rounds of algebraic manipulations,

$$
(\phi - \mu) \{ \frac{\partial Q_e}{\partial K_e} [\alpha \beta + r(1 - \alpha)] - \beta r \} = 0.
$$

Since, $\partial U/\partial C_m = \phi - \mu > 0$, the first result in Proposition 2 follows.

Next, turn to the home goods sector. Set $J_h$ equal to zero. Assuming an interior solution, from (B.1) and (B.3), $\lambda_h = \partial U/\partial C_m$. Set $\dot{\lambda}_h = 0$, and substitute $\lambda_h = \partial U/\partial C_m$ to get the second result in Proposition 2.

**B.2 Proof of Proposition 3.** From proposition 2, $K_e^*$ solves,

$$
\frac{\partial Q_e}{\partial K_e^*} = \frac{\beta r}{r(1 - \alpha) + \alpha \beta}.
$$

(B.10)
Assuming that the debt/export and the vulnerability ratios are equal in the long-run, $D^*$, satisfies,

$$rD^* = \alpha Q_e(K_e^*)^\beta.$$  \hspace{1cm} (B.11)

The long-run foreign goods consumption is given by,

$$C_m^* = Q_e(K_e^*) - rD^*.$$ \hspace{1cm} (B.12)

Finally, the home goods capital stock in the long-run is obtained from,

$$\frac{\partial U}{\partial C_m^*} = \frac{\partial U}{\partial Q_h(K_h^*)} \frac{\partial K_h^*}{\partial \alpha},$$ \hspace{1cm} (B.13)

where $W_2 = 0$ is assumed\textsuperscript{11}.

Differentiating (B.10), (B.11), (B.12), and (B.13) and assuming home and foreign goods to be gross substitutes, we get the comparative static results in Proposition 3:

\[
\frac{\partial K_e^*}{\partial \alpha} = -\beta r(\beta - r)/[(r + \alpha(\beta - r))^2 \partial^2 Q_e/\partial K_e^2] \geq 0,
\]

\[
\frac{\partial D^*}{\partial \alpha} = \frac{1}{r}[Q_e + \alpha \partial Q_e/\partial K_e^*] > 0,
\]

\[
\frac{\partial C_m^*}{\partial \alpha} = -Q_e + (1 - \alpha) \frac{\partial Q_e}{\partial K_e^*} \frac{\partial K_e^*}{\partial \alpha} < 0,
\]

\[
\frac{\partial K_h^*}{\partial \alpha} = \frac{\partial C_m^*}{\partial \alpha} \frac{\partial Q_h}{\partial K_h} \frac{\partial^2 U}{\partial K_h \partial C_h} - \beta \frac{\partial^2 U}{\partial C_m^2} \frac{\partial U}{\partial C_h}^2 < 0.
\]

\textsuperscript{11}If $K_h^*$ turns out to be less than the initial capital stock in the home goods sector, the stationary value of $W_2$ is then: $W_2 = (\partial U/\partial C_h) - \{(\partial U/\partial C_h)(\partial Q_h/\partial K_h)\}/\beta$.  

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References


